5/3/18

"Its easy to be happy when people do what they're supposed to."- Chris Callahan

HW: "Direct Variation" Homework section

AIM: What is Direct Variation?

Warm Up:

Determine the equation of the parabola whose focus is the point (4, 1) and whose directrix is the horizontal line $y = -3$. First, draw a diagram that shows the parabola, then carefully use the distance formula to derive its equation.

Vertex:

$\frac{1 + -3}{2} = -1$

Vertex: $(4, -1)$

P-value:

$\frac{1-(-3)}{2} = 2$

$y = \frac{1}{4(2)}(x-4)^2 + (-1)$

$y = \frac{1}{8}(x-4)^2 - 1$

Alt:

Distance to directrix = Distance to Focus

$y = -3$

Focus: $(4, 1)$

$y - (-3) = \sqrt{(x-4)^2 + (y-1)^2}$

$(y+3)^2 = \left(\sqrt{x^2-8x+16+y^2-2y+1}\right)^2$

$y^2+6y+9 = x^2-8x+16+y^2-2y+1$

$-y^2+2y-9 \quad\quad -y^2+2y-9$

$\frac{8y}{8} = \frac{x^2}{8} - \frac{8x}{8} + \frac{8}{8}$

$y = \frac{1}{8}x^2 - x + 1$

$(x-4)^2 = (x-4)(x-4)$

$x^2 - 4x - 4x + 16$

$x^2 - 8x + 16$

PROPORTIONAL OR DIRECT RELATIONSHIPS

Two variables, x and y, have a **direct (proportional) relationship** if for every ordered pair (x, y) we have:

$$\frac{y}{x} = k \text{ or } y = kx$$

Stated succinctly, y will always be a constant multiple of x. The value of k is known as the **constant of variation**.

***Exercise* #1:** In each of the following, x and y are directly related. Solve for the missing value.

(a) $y = 15$ when $x = 5$

$y = ?$ when $x = 9$

$y = 27$

$\frac{y}{x}$

$\frac{15}{5} = 3 = \frac{y}{9}$

$\frac{15}{5} = \frac{y}{9}$

$\frac{5y}{5} = \frac{135}{5}$

$y = 27$

(b) $y = -6$ when $x = 4$

$y = ?$ when $x = -10$

$\frac{y}{x}$ $\frac{(-10)}{} - \frac{6}{4} = \frac{y}{-10}$ (-10)

$\frac{+60}{4} = 15 = y$

$\frac{-6}{4} = \frac{y}{-10}$

$\frac{60}{4} = \frac{4y}{4}$

$y = 15$

(c) $y = 12$ when $x = 16$

$y = ?$ when $x = 24$

$\frac{y}{x}$

$\frac{12}{16} = \frac{y}{24}$

$y = 18$

$\frac{288}{16} = \frac{16y}{16}$

***Exercise* #2:** The distance a person can travel varies directly with the time they have been traveling if going at a constant speed. If Phoenix traveled 78 miles in 1.5 hours while going at a constant speed, how far will he travel in 2 hours at the same speed?

$\frac{\text{miles}}{\text{hrs}}$ $\frac{78}{1.5} = \frac{x}{2}$

$\frac{156}{1.5} = \frac{1.5x}{1.5}$

$104 = x$

104 miles

***Exercise* #3:** Jenna works a job where her pay varies directly with the number of hours she has worked. In one week, she worked 35 hours and made $274.75. How many hours would she need to work in order to earn $337.55?

$\frac{\text{money}}{\text{hours}}$ $\frac{274.75}{35} = \frac{337.55}{x}$

$\frac{11814.25}{274.75} = \frac{274.75x}{274.75}$

$x = 43$ hours

Exercise #4: Two variables, x and y, vary directly. When $x = 6$ then $y = 4$. The point is shown plotted below.

(a) Find the y-values for each of the following x-values. Plot each point and connect.

$x = 3$ (3,2)

$\frac{x}{y}$ $\frac{3}{y} = \frac{6}{4}$

$12 = 6y$

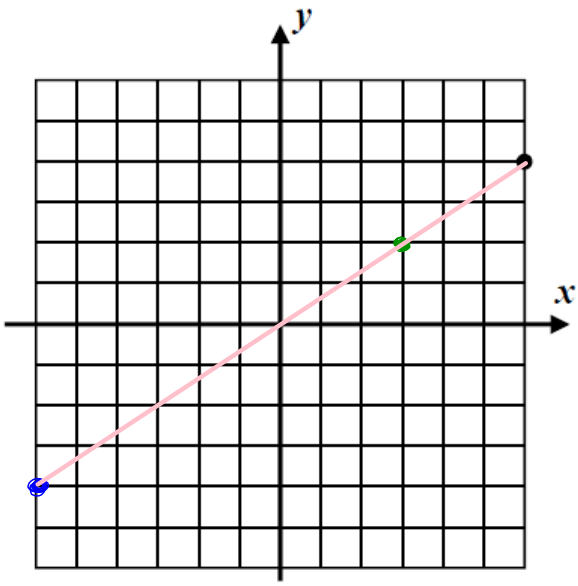
$2 = y$

$x = -6$ (-6,-4)

$\frac{x}{y}$ $\frac{-6}{y} = \frac{6}{4}$

$-24 = 6y$

$-4 = y$



(b) What is the constant of variation in this problem? What does it represent on this line?

$\frac{y}{x} = k$

$\frac{4}{6} = \frac{2}{3} = \frac{-4}{-6} = \boxed{\frac{2}{3}}$

k is slope of line

(6,4) (3,2) (-6,-4)

(c) Write the equation of the line you plotted in (a).

$(y - y_c) = m(x - x_c)$

$y - 4 = \frac{2}{3}(x - 6)$ ← Same → $\boxed{y = \frac{2}{3}x}$

$y = mx + b$

$y = \frac{2}{3}x + 0$

Slope $= \frac{2}{3}$

$b = 0$

Direct relationships often exist between two variables whose values are zero simultaneously.

Exercise #3: The miles driven by a car, d, varies directly with the number of gallons, g, of gasoline used. Abagail is able to drive $d = 336$ miles on $g = 8$ gallons of gasoline in her hybrid vehicle.

(a) Calculate the constant of variation for the relationship $\frac{d}{g}$. Include proper units in your answer.

$$\frac{d}{g} = \frac{336 \text{ miles}}{8 \text{ gallons}} = 42 \frac{\text{miles}}{\text{gallon}}$$

(b) Give a linear equation that represents the relationship between d and g. Express your answer as an equation solved for d.

$$(g)\ \frac{d}{g} = 42(g)$$

$$\boxed{d = 42g}$$

(c) How far can Abagail drive on $g = 6$ gallons of gas?

$$d = 42(6)$$

$$\boxed{d = 252 \text{ miles}}$$

(d) How many gallons of gas will Abagail need in order to drive 483 miles?

$$\frac{483}{42} = \frac{42g}{42}$$

$$\boxed{11.5 \text{ gallons}}$$